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**Algebra 2H Unit “8”: Trigonometric Functions and The Unit Circle --- Notes Day 1**

So, what is Trigonometry? At its core, Trig (that’s a simpler abbreviation that I will use quite often) is the study of the relationships between sides and angles of a triangle. I know that it sounds like Geometry, and there are a lot of tie-ins to Geometry, but it is its own branch of mathematics. We could spend almost an entire year on just investigating Trig and its relationships with both Algebra and Geometry.

There are two main types of Trig: 1) Right Triangle Trigonometry and 2) Non-Right Triangle Trigonometry. We will only be focusing on the first type, involving right triangles. You may be exposed to some non-right triangle stuff once you reach pre-calculus and beyond. But what we do in this unit will serve as a basis for all of that.

In Algebra 1, I am sure you spent a little time with right triangles, specifically the Pythagorean Theorem (which most of you memorized as: a2 + b2 = c2). Remember that the “a” and “b” values refer to the legs of the right triangle, the two sides that come together to form the right angle. The “c”-value is the hypotenuse, which is always directly across from the right angle. The “a” and “b” values are interchangeable in the equation, but the “c”-value must always represent the hypotenuse. The picture below should refresh your memory.

a

b

c

To help illustrate the relationships between the sides and angles, we need to be introduced to three new functions: Sine, Cosine, and Tangent. These are often abbreviated as: Sin, Cos, Tan and these buttons can be found on scientific calculators (even your iPhones if you turn them sideways!) and on graphing calculators like the Inspire.

Each of the three trig functions (Sin, Cos, Tan) represent different side relationships based on a given angle. In trig, you may notice the use of Greek letters to represent angles. Where we might use “x” or “n”, trig uses It’s not necessary to know the Greek alphabet, just be ready to see these new symbols as we move forward. I tend to use a lot when representing angles…just a habit I developed over many (30+) years of doing this. Angles may also be referred to by their letter.

Are you ready to take on a new challenge…a bit nervous…interested (I hope)? Then here we go…

**Part One --- Defining the three Trigonometric Functions (Sin, Cos, Tan)**

To understand Trigonometry, we really have to know the three main functions. Each one represents the ratio of sides of a triangle, and in a specific order. The triangle below will help us to understand how each trig function relates to a pair of sides. We will use to represent an unknown angle of the triangle.

Hypotenuse (H)

Opposite Side (O)

Adjacent Side (A)

The Hypotenuse (which we will simply call H) is always the longest side and right across from the right angle. The Opposite side (O), will vary depending on where is. If you draw a straight line out from , the side you intersect is considered opposite (O) from it. The Adjacent side (A) is the remaining side, and it is always between the angle () and the right angle.

I know, I know…but what does this have to do with Sin, Cos, and Tan? It has to do with an old horse…

Some Old Horse Caught Another Horse Taking Oats Away

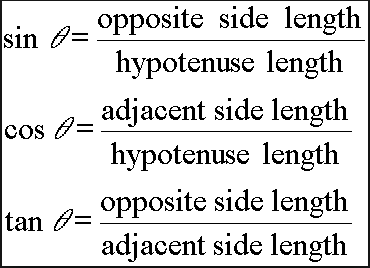
Now I know that some of you (most of you?) are rolling your eyes and making fun of me…go ahead, laugh it up! This corny little phrase will be stuck in your head for the rest of your life. Ask your parents if they know SOHCAHTOA or some phrase about a horse…I’ll wait…

Bet many of your parents sighed or rolled THEIR eyes, but they still remembered it! This phrase is how we remember which trig function goes with which pair of sides.

Some Old Horse Caught Another Horse Taking Oats Away

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Sine Opposite Hypotenuse Cosine Adjacent Hypotenuse Tangent Opposite Adjacent

The first letters of each word form a pneumonic that helps us remember the side relationship for each function.

Is the confusion setting in yet? I figured…let’s try an example to illustrate the functions and see if that helps. The right triangle below has all three side lengths given.

5

12

13

Based on the lengths as given, and SOHCAHTOA (that’s the trig relationships), here’s how we should attack this:

1) label the three sides with O (Opposite), A (Adjacent) and H (Hypotenuse) with respect to angle

2) Find the value (ratio) of the three trig functions

So, adjusting the diagram above, it would now look like this:

5

12

13

Hypotenuse (H)

Opposite Side (O)

80

Adjacent Side (A)

So, here are our three trig ratios for this problem:

Each trig function is represented as the ratio of two sides, depending on where lies. Please realize that doesn’t always have to be in the lower angle. Which means that the O and A can change depending. The H will always remain across from the right angle.

**Trig 2020 – Notes Day #2**

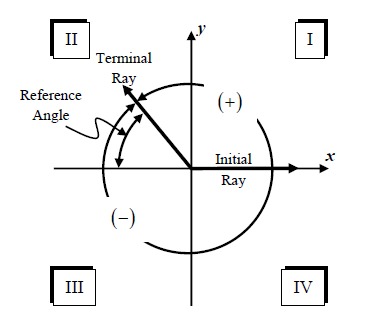
Okay, so we introduced the main three trig functions that we will be working with this unit. These functions are based on the geometry of a circle and rotations around its center. Sometimes the trigonometric functions are known as **circular functions.** In this next lesson, we introduce some basic terminology and concepts concerning angles.

**Standard Position:** An angle is said to be drawn in standard position if its vertex is at the origin and its initial ray points along the positive *x*-axis.

**Positive and Negative Rotations:** A rotation is said to be ***positive*** if the initial ray is rotated counterclockwise to the terminal ray and said to be ***negative*** if the initial ray is rotated clockwise to the terminal ray. It may seem counterintuitive that clockwise is negative, but that’s the way it is.

**Coterminal Angles:** Any two angles drawn in standard position that share a terminal ray.

**Reference Angles:** The positive acute angle formed by the terminal ray and the ***x*-axis**.



90

180

0

270

Now let’s try a few practice questions:

*Hint*: By either adding (counterclockwise) or subtracting (clockwise) we travel a full rotation, thus ending on the same terminal ray (coterminal angles).

Example 1 For each of the following angles, given by the Greek letter **theta**, , identify the quadrant that the terminal ray falls in. You can use roman numerals or standard Arabic.

1. (b) (c)

145 is between 90 and 180, 320 is between 270 and 360, -210 is between -180 and -270,

so it lies in Quad. 2 so it lies in Quad. 4 so it lies in Quad. 2

(d) (e) (f)

72 is between 0 and 90, 250 is between 180 and 270, -460 = -100, and is between -90 and

so it lies in Quad. I so it lies in Quad. III -180, so it lies in Quad. III

**The negative angles are always harder! Just remember that each group of 360 is one full rotation.**

Example 2 In which quadrant would the terminal ray of an angle drawn in standard position fall if the angle

measures ?

1. I (3) III If you subtract two groups of 360 (so, 720), you’d have
2. II (4) IV 140…which would fall in Quad. 2

Example 3 Give a negative angle that is coterminal (lands in the exact same spot) with each of the following positive angles, given by the Greek letter **alpha**, .

1. (c)

-270 -30

1. (d)

-240 -150

Do you notice a pattern with the two numbers?

Example 4 Coterminal angles drawn in standard position will always have measures that differ by an integer

multiple of

1. (3)
2. (4)

Example 5 For each of the following angles given by the Greek letter **beta**, , state **beta’s** reference angle, . Remember that a reference angle is the distance from that angle to the x-axis. Reference angles are always positive and acute (less than 90).

1. (b) (c)

180 – 160 = 20 360 – 300 = 60 210 – 180 = 30

(d) (e) (f)

Since it’s acute, it is 180 + (-110) = 70 360 + (-280) = 80

already a ref angle.

**Now, try the questions on Trig Assignment #2.**